Topics in Graph Theory
MATH 4330/5330
Winter 2020

LECTURES: MF 1:05–2:25, LSC-Psychology 5208 (Subject to change). Web presence: on BrightSpace.

INSTRUCTOR: Dr. Jeannette Janssen.

- Office: Chase building, room 315.
- Office hours: Wednesday 2.30-3.30pm, or by appointment.
- Email: Jeannette.Janssen@dal.ca, or use the email function in BrightSpace.

Course Description: This is a topics course intended for math and computer science students. Topics this term will focus on graph colouring, graphs and optimization, and graph clustering.

Course Prerequisites: MATH3330, CSCI 3110 or permission from instructor

Course Text: There is no course text. Class notes and background reading material will be posted on Brightspace. Basic knowledge of graph theory is expected; for background, I recommend consulting one of the following text books (available in the Killam library):

- Introduction to Graph theory. Douglas West
- Graph theory. Richard Diestel.
- Graph theory and its Applications. Gross and Yellen.

I will also provide links to free on-line texts.

Other relevant policies A document containing Dalhousie's policies on plagiarism, accessibility, and a number of other important issues has been posted to Brightspace.

Evaluation:

There will be slight differences in the evaluation and requirements for students taking this course at the graduate level (MATH 5330) or undergraduate level (MATH 4330/CSCI 4115).

Assignments. The evaluation of this course is based in large part on the assignments. While you are encouraged to work together with your class mates, the final write-up of the assignment must be done individually; please read the university plagiarism policy, that can be found on the course Web page. Any material consulted must be properly credited; this includes web pages, or personal communication. For students taking this class as a graduate class (MATH 5330), some of the assignment problems will differ.

Exams. There will be two exams. The first will be an in-class exam, held on Friday, Feb. 28. The second will be a take-home exam, given out at the end of term. MATH 5330 students will only take the in-class exam.

Short presentations (MATH 4330/CSCI 4115 only). Undergraduate students will be expected to give a 10 minute presentation at some point during the term. Here the student will present a theorem relevant to the class material (chosen from a given list), and explain its proof. In addition, the student will provide a write-up, which will be posted on Brightspace.

Term project (MATH 5330 only). Graduate students will be expected to do a term project. This consists of reading and digesting a scientific paper on a topic related to the class (chosen from a given list). Students will give a (20 minute) presentation on their topic in class towards the end of term, and produce a report of 10–15 pages.

MATH 5330	Assignments:	40%
	Midterm exam:	30%
	Project :	30%
MATH 4330/ CSCI 4115	Assignments:	40%
	Presentation:	10%
	Midterm exam:	30%
	Take-home exam :	20%

Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale:

A+ (90–100)	B+ (77–79)	C+ (65–69)	D (50–54)
A (85–89)	B (73–76)	C (60–64)	F (< 50)
A- (80–84)	B- (70–72)	C- (55–59)	

Topics:

(1) Graph colouring and related topics: chromatic number and index, independence number and clique number, lower and upper bounds, k-degenerate graphs, interval graphs, list colouring.

(2) Graphs and optimization. Fractional colouring. Perfect graphs. Fractional equivalents of graph parameters: matching number, domination number. Formulating optimization problems as integer linear programs. Using LP duality to bound such graph parameters.

(3) Graph clustering and graph similarity. Homomorphisms to classify graph structure. Spectral graph theory. The Laplacian of a graph, the stochastic block model. Modularity and clustering coefficient. Community detection in large networks.

Course Objectives/Learning Outcomes

- Know the definitions of vertex and edge colouring, chromatic number and index, independence number and clique number, and be able to give the relations between these different concepts
- Be able to prove basic results regarding the chromatic number of certain classes of graphs.
- Know the greedy algorithm for colouring the vertices of a graph, as applied to k-degenerate graphs and proper interval graphs.
- Be able to formulate graph-theoretic combinatorial optimization problems (finding the largest independent set, finding the maximum matching, etc.) as an Integer Program.
- Given an Integer Program representing a combinatorial optimization program, be able to interpret the fractional solutions to the Linear Programming Relaxation.
- Know the LP Duality theorem, and be able to apply in order to obtain lower bounds for Combinatorial Optimization problems.
- Know how to form and interpret the adjacency matrix and Laplacian matrix of a graph.

- Be able to comment on and evaluate the main approaches to graph clustering.
- Be able to present the graph clustering problem in terms of the adjacency and Laplacian matrix of a graph.